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Lancio di 2 monete non truccate.

$T \to 1$

$C \to 0$

$\Omega = \{(1,1); (1,0); (0,1); (0,0)\}$

spazio campione

$(1,1) \in \Omega$ è un evento elementare

$\omega \in \Omega$

$$P(\{\omega\}) = P(\omega)$$

$$P((0,0)) = \frac{1}{4}$$

$P($esca almeno una testa$) =$

$$= P(\{(1,0), (0,1), (1,1)\}) =$$

$$= P(\{(1,0)\} \cup \{0,1\} \cup \{1,1\})$$

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$\Omega = \{1, \dots, 90\}$

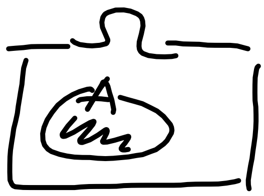
2 R 4 B

$\Omega = \{R, B\}$

Dato un esperimento aleatorio

- Trovare lo spazio campione Ω
- Definire una famiglia di sottoinsiemi di Ω che riteniamo interessanti
- Definire su questa famiglia la Probabilità

DEFINIZIONE: Dato un insieme Ω chiamiamo σ-algebra $\mathcal{A}$ una famiglia di sottoinsiemi di Ω tale



i) $\Omega \in \mathcal{A}$

ii) Se $A \in \mathcal{A} \Rightarrow A^c \in \mathcal{A}$

iii) Se $(A_n)_n \subset \mathcal{A} \Rightarrow \bigcup_n A_n \in \mathcal{A}$

$$(A_n)_n \subset \mathcal{A} \Rightarrow (A_n^c)_n \subset \mathcal{A}$$

$$\bigcup A_n \in \mathcal{A}$$

$$A \cap B = (A^c \cup B^c)^c$$
$$\| $$
$$(A^c)^c \cap (B^c)^c$$

$$\bigcap A_n = \left(\bigcup A_n^c\right)^c$$

$$\mathcal{A} \ni \left(\bigcup A_n^c\right)^c = \bigcap (A_n^c)^c = \bigcap A_n \in \mathcal{A}$$

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$\{\Omega, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{1\}^c, \{2\}^c \ldots$

$\ldots$

$\mathcal{A} = \mathcal{P}(\Omega)$

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$\Omega, \mathcal{A}$. La probabilità è una funzione

$$P : \mathcal{A} \longrightarrow \mathbb{R}^+$$
$$A \longrightarrow P(A)$$

tale che

i) $P(\Omega) = 1$

ii) $\forall (A_n)_n \subset \mathcal{A}$ t.c. $A_i \cap A_j = \emptyset \ \forall i \neq j$ valere

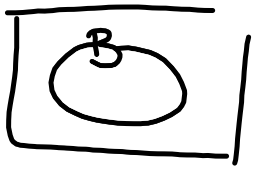
$$P(\cup A_n) = \sum_n P(A_n)$$

$(\Omega, \mathcal{A}, P) \rightarrow$ spazio probabilizzato

- Ω evento certo
- $\emptyset$ $P(\emptyset)$? evento impossibile

Conseguenze degli assiomi.

$(\Omega, \mathcal{A}, P)$



$\forall A, B \in \mathcal{A}$

$$B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B) \quad (*)$$

$$(A \cap B) \cap (A^c \cap B) = \emptyset .$$

perché $A \cap A^c = \emptyset$

a) $B = \Omega$

$\Omega = A \cap \Omega \cup A^c \cap \Omega = = \underline{A} \cup \underline{A^c}$

$P(\Omega) = P(A) + P(A^c)$

$\overset{\shortparallel}{1}$

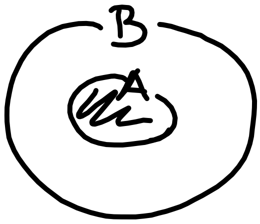
$\forall A \in \mathcal{A} \quad P(A) + P(A^c) = 1$

$P(A^c) = 1 - P(A)$

2) $A, B \in \mathcal{A}$ $\quad \underline{A \subseteq B} \Rightarrow P(A) \leq P(B)$ (isotonia)

$$P(B) = P(A \cap B) + P(A^c \cap B) =$$

$$= P(A) + P(A^c \cap B) \geq P(A)$$

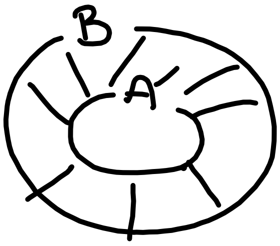


3) $\forall A \in \mathcal{A}$ $\quad P(A) \leq 1$

$$A \subseteq \Omega \Rightarrow P(A) \leq P(\Omega) = \underline{1}$$

4) $A \subseteq B$ posso scrivere

$$A^c \cap B = B \setminus A$$



$$P(B) = P(A \cap B) + P(A^c \cap B) =$$

$$= P(A) + P(B \setminus A)$$

$$P(B \setminus A) = P(B) - P(A)$$

3) A, B qualsiasi

$$A \cup B = A \cup (A^c \cap B) \Rightarrow A \cap (A^c \cap B) = \emptyset$$

$$P(A \cup B) = P(A) + P(A^c \cap B) =$$

$$= P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) + \\ - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ + P(A \cap B \cap C)$$

$\{A_n\}_{n\in\mathbb{N}} \subset \mathcal{A}$

$$\left(\bigcup A_n\right)^c = \bigcap A_n^c$$

$$P\left(\bigcup A_n\right)^c = 1 - P\left(\bigcap_n A_n^c\right)$$

$$\bigcap_n A_n^c \neq \left(\bigcap A_n\right)^c$$

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3 rosse 2 bianche

$\Omega = \{r, b\}$ $\mathcal{A} = \mathcal{P}(\Omega)$

$P(\{r\}) = \frac{3}{5}$ $P(\{b\}) = \frac{2}{5}$

$\Omega = \{r_1, r_2, r_3, b_1, b_2\}$ $P(\{r\}) = P(\{r_1\} \cup \{r_2\} \cup \{r_3\}) =$

$= P(\{r_1\}) + P(\{r_2\}) + P(\{r_3\}) =$

$= \frac{3}{5}$

3 r   2 b

1

r_1 — r_2, r_3, b_1, b_2

r_2 — r_1, r_3, b_1, b_2

r_3 — r_1, r_2, b_1, b_2

b_1 — r_1, r_2, r_3, b_2

b_2 — r_1, r_2, r_3, b_1

Casi possibili 20

Casi favorevoli 12

P(esce una rossa alla seconda estrazione) =

$$= \frac{12}{20} = \frac{3}{5}$$

$\boxed{(A_n)_n}$ $\forall n \; A_n \in \mathcal{A}$

$\{A_n\}_{n\in\mathbb{N}}$ $\{A_1, A_2, A_3, \;, A_n, \ldots \ldots\}$

$\{A_n\}_{n\in\mathbb{N}}$ $\{A_i\}_{i=1\ldots7}$

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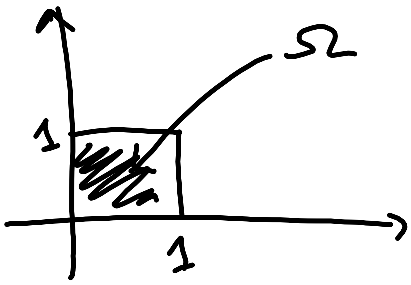

$\mathcal{A}$ = sottoinsiemi mis. di Ω

$A \subset \Omega$ $\quad P(A)$ = area di A